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J.J.M. Too and Xuejing Feng
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A NUMERICAL METHOD FOR THE PREDICTION OF INHOMOGENEOUS DEFORMATION IN ROLLING PROCESSES

by

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ABSTRACT

A numerical method for the prediction of the inhomogeneous deformation of a rolling process, based on the control volume method and flow theory is presented. The temperature and rate dependency, the coupled thermal/mechanical behaviour involving heat conduction and heat generation from plastic energy dissipation are accounted for. The model is capable of predicting temperature distribution, and rate of deformation during a hot/cold rolling process.

LABORATOIRES DE RECHERCHE MÉTALLURGIE PHYSIQUE
RAPPORT LRMP 85-34(TR)

MÉTHODE NUMÉRIQUE POUR PRÉVOIR LES VARIATIONS DE DÉFORMATION LORS DU LAMINAGE

par

J.J.M. Too et Xuejing Feng

RÉSUMÉ

Les variations de la déformation lors du laminage sont prévues par une méthode numérique basée sur le contrôle du volume et la théorie de l'écoulement. On considère l'effet de la température et de la vitesse et le comportement thermomécanique tenant compte de la conduction thermique et de la génération de chaleur par dissipation de l'énergie plastique. Le modèle peut prédire la distribution de la température et la vitesse de déformation durant le laminage à chaud et à froid.

INTRODUCTION

In rolling processes, it has been well recognized that the deformation behaviour is a strong function of temperature and rate of deformation. These factors have significant effects on the quality of rolled products, which call for appropriate design of rolling reduction and temperature schedules. Technology in the design of a rolling schedule can be greatly enhanced if the distribution of temperature and deformation can be predicted correctly by models.

In a previous paper by J.J.M. Too et al. (1), a finite element model based on the finite strain theory was presented. The model works well and its results compared well with experimental data. However, it requires a large amount of computing time. While it offers a useful analytical tool for the understanding of the deformation mechanism during rolling, its demand for excessive computation effort makes the model impractical for direct application to the design of rolling schedules and on-line control of rolling processes. These factors prompted the development of a simplified method that is practical for the design of rolling schedules. Such a model requires the following features and capabilities:

- Prediction of coupled thermal/mechanical behaviour involving plastic deformation and heat conduction, as well as heat generation from plastic energy dissipation.
- Minimal computing time.
- Application to the design of rolling reduction and temperature schedules.
- Possible application to on-line control of rolling processes.

Normally, mathematical models for the prediction of metal deformation during rolling are based on solid mechanics, i.e., the workpiece is treated as solid. In this work, the workpiece is considered as a viscous fluid as it passes through the roll gap. The formulation is thus derived based on viscous flow theory. The equations are solved by either the finite difference method or the control volume method. Both methods are available in the computer model. However, only the control volume method will be reported.

Results obtained from the model indicate that coupled thermal/mechanical behaviour can be adequately modelled, and that the heat production from plastic deformation can also be predicted.

As expected, the present approach resulted in a substantial saving in computation in contrast to the finite element model based on finite strain theory. It offers therefore the possibility for use in the on-line control of rolling processes.

PROBLEM FORMULATION

The inhomogeneous nature of plastic deformation was first treated by Orowan (2). It has since been investigated by various researchers (for example, Ref. 3-5). However, only recently has the inclusion of coupled thermal problems been considered by Kobayashi (6), Zienkiewicz et al. (7), Argyris et al. (8) and Too et al. (1) using finite element methods.

In this report, we have adapted the control volume method based on viscous flow theory. In the process of formulating the coupled thermal/mechanical problem of rolling, the following assumptions are made for the sake of simplification in order to achieve minimum computing time:

- The workpiece is assumed to be very long and wide so that a 2-dimensional plane-strain approximation is applicable (the case in industrial rolling).
- Elastic deformation is negligible.
- A steady-state condition is operative because of the large length of the workpiece relative to the contact length with the roll.
- The temperature of the roll is constant as a consequence of the steady-state assumption.

A schematic description of the rolling process is shown in Fig. 1. As we have already mentioned, in hot/cold rolling, the deformation process is highly coupled with heat transport and energy dissipation. This coupled behaviour must be considered in the formulation of the model.

First consider the deformation process. It can be described mathematically by the incompressible flow equations derived from the principle of conservation of momentum continuity conditions:

$$\rho \frac{\partial u}{dt} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial x}\left(\mu \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \qquad \text{Eq 1}$$

and

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{Eq 2}$$

In the above equation, u and v denote the velocity components in the x and y directions, respectively, ρ is the density of the workpiece and μ is the nonlinear viscosity of the material which is a function of temperature and strain rate. The first term in Eq 1 vanishes due to the steady-state assumption. Equation 1 then becomes:

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial x}\left(\mu \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \qquad \text{Eq 1a}$$

The boundary conditions for the problem under investigation are:

$$v = 0, \text{ and } \frac{\partial u}{\partial y} = 0 \text{ at } y = 0$$

This is because of the symmetrical nature of the problem with respect to the centre plane of the workpiece. The same boundary conditions apply at the sections remote from the entrance and exit because no surface traction exists before the entrance and beyond the exit. These boundary conditions are depicted in Fig. 2.

The heat generated from plastic energy dissipation can be expressed as:

$$Q = \frac{f}{J} \int \sigma \cdot \dot{\varepsilon} \, dA \qquad \text{Eq 3}$$

where σ and $\dot{\varepsilon}$ are the stress and rate of strain respectively; J is the mechanical equivalent of heat; and f is the fraction of the plastic energy dissipated in the form of heat.

This heat production, Q, must be incorporated in the heat transport equation:

$$\rho c \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + Q \qquad \text{Eq 4}$$

where c is the specific heat and k is the thermal conductivity of the media. Note that the first term of Eq 4 also becomes zero for a steady-state condition. Equation 4 can thus be written as:

$$\rho c \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + Q \qquad \text{Eq 4a}$$

The boundary conditions for the thermal diffusion-convection considered are:

$$k \frac{\partial T}{\partial n} = h_f (T - T_r) \quad \text{at the contact interface}$$

$$k \frac{\partial T}{\partial y} = h_a (T - T_a) \quad \text{at the convective boundary,}$$

and $$\frac{\partial T}{\partial y} = 0 \quad \text{at } y = 0,$$

where h_f and h_a are heat transfer coefficients at the interface and the convective boundary as indicated in Fig. 3, T_r and T_a are the roll and ambient temperatures, respectively and n is the normal at the contact interface.

The similarity of Eq 1a and 4a may be utilized to advantage when solving these equations. If u in Eq 1a and T in Eq 4a are replaced by a general field function ψ, and the coefficients μ and k by a general notation Γ, Eqs 1a and 4a represented by one expression:

$$L(\psi) = \rho' u \frac{\partial \psi}{\partial x} + \rho' v \frac{\partial \psi}{\partial y} - \frac{\partial}{\partial x} \Gamma \frac{\partial \psi}{\partial x} - \frac{\partial}{\partial y} \Gamma \frac{\partial \psi}{\partial y} - Q = 0 \qquad \text{Eq 5}$$

Then the same solution algorithm can be used for both the incompressible flow problem and the heat conduction problem. To solve this problem we have used the control volume method which will be discussed in the next section.

NUMERICAL MODEL

The weighted residual expression for Eq 5 can be written as:

$$\int_{\Omega} w\, L\,(\bar{\Psi})\, d\Omega = 0 \qquad \text{Eq 6}$$

where w is the weighting function, Ω represents the domain under study, and $\bar{\Psi}$ is the approximate solution of Eq 5. The control volume method happens to have the simplest form of weighting function (w = 1). Equation 6 thus becomes:

$$\int_{\Omega} L\,(\bar{\Psi})\, d\Omega = 0 \qquad \text{Eq 7}$$

A number of weighted residual equations can be generated by dividing the problem domain into subdomains as shown in Fig. 4, and setting the weighting function to unity over one subdomain, and zero elsewhere. It should be noted, as shown by Eq 7, this method automatically satisfies the conservation principle which is one of the important features necessary for better solution convergence.

The control volume around a mesh point (i,j), for a 2-D case, is the shaded area bounded by the dashed lines, as shown in Fig. 4.

The integration of Eq 7 over the control volume leads to an expression referring to locations e, w, n and s which are, in this case, the mid points of two consecutive mesh points.

$$(\rho' u\, \bar{\Psi} - \Gamma \frac{\partial \bar{\Psi}}{\partial x})_e - (\rho' u\, \bar{\Psi} - \Gamma \frac{\partial \bar{\Psi}}{\partial x})_w + (\rho' v\, \bar{\Psi} - \Gamma \frac{\partial \bar{\Psi}}{\partial y})_n$$

$$- (\rho' v\, \Psi - \Gamma \frac{\partial \bar{\Psi}}{\partial y})_s = Q\Delta x\Delta y \qquad \text{Eq 8}$$

or

$$J_e - J_w + J_n - J_s = Q\Delta x\Delta y \qquad \text{Eq 9}$$

where J_e, J_w, J_n and J_s are the total convective and diffusive fluxes as indicated in Fig. 4.

Similarly, integrating the continuity equation (Eq 2), we obtain:

$$(\rho u \Delta y)_e - (\rho u \Delta y)_w + (\rho v \Delta x)_w - (\rho v \Delta y)_s = 0 \qquad \text{Eq 10}$$

$$F_e - F_w + F_n - F_s = 0 \qquad \text{Eq 11}$$

combining Eq 9 and Eq 11 yields

$$(J_e - F_e \bar{\Psi}_{i,j}) - (J_w - F_w \bar{\Psi}_{i,j}) + (J_n - F_n \bar{\Psi}_{i,j}) - (J_s - F_s \bar{\Psi}_{i,j}) = Q \Delta x \Delta y \qquad \text{Eq 12}$$

Rearranging Eq 12 to solve for $\bar{\Psi}_{i,j}$, the final discretized equation set can be written as:

$$A_{i,j} \bar{\Psi}_{i,j} = A_{i+1,j} \bar{\Psi}_{i+1,j} + A_{i-1,j} \bar{\Psi}_{i-1,j} + A_{i,j+1} \Psi_{i,j+1} + A_{i,j-1} \Psi_{i,j-1} + B \qquad \text{Eq 13}$$

where $A_{i+1,j} = (\frac{1}{2} \rho u_e - \frac{\Gamma e}{\Delta x}) \Delta y$

$A_{i-1,j} = (\frac{1}{2} \rho u_w - \frac{\Gamma w}{\Delta x}) \Delta y$

$A_{i,j+1} = (\frac{1}{2} \rho v_n - \frac{\Gamma n}{\Delta y}) \Delta x$

$A_{i,j-1} = (\frac{1}{2} \rho v_s - \frac{\Gamma s}{\Delta y}) \Delta x$

$A_{i,j} = A_{i+1,j} + A_{i-1,j} + A_{i,j+1} + A_{i,j-1}$

$i,j = 1 \sim n$ and $1 \sim m$ respectively

n,m = total no. of mesh points in x, and y direction respectively

$B = Q \Delta x \Delta y$

The upwinding scheme was used to obtain these solutions.

SAMPLE PROBLEM

To demonstrate the validity of the present model, a hot rolling problem as depicted in Fig. 5 is analyzed using the model.

The initial height h_o of the workpiece is 117.0 mm. The roll gap h_i at the exit is 100.3 mm. The roll, measuring 228.5 mm in diameter, is rotating with a speed of 1016 mm/s. The temperature of the roll is assumed to be 400°C. The workpiece is preheated to 1200°C before rolling.

In this analysis, a sticking condition at the roll/workpiece interface is assumed for simplicity, although a slipping condition can also be accounted for by the model. The analysis is carried out based on the discretized mesh points shown by Fig. 6. Results predicted by the model are plotted in Fig. 7 and Table 1, at cross sections indicated in Fig. 6.

Figure 7 shows the horizontal and vertical velocity profiles at sections indicated in Fig. 6. Clearly, the inhomogeneous deformation of a rolling process is correctly modelled. It is of interest to note that, as expected, the velocity of the plastic flow is larger at and near the contact interface than at the center of the workpiece. At the exit, the velocity profile of material flow approaches a uniform distribution across the thickness. This is indeed the case in a real rolling process, because there is no surface traction applied on the surface of the workpiece beyond the point of the exit.

The temperature distributions at sections between entry point and exit are tabulated in Table 1. The expected large temperature gradients at and near the contact interface are again correctly predicted.

Two different analyses were carried out for this sample problem. In one of the analyses, the heat production from plastic energy dissipation were accounted for (with DQ). The other is analyzed without considering the heat

production (no DQ). It is important to point out that the former has predicted a maximum of 50°C increase from plastic energy dissipation for the case studied. This temperature increase must be accounted for in the design of rolling schedules, especially when rolling is conducted near phase transformation temperatures. The model described here has such capability as indicated by the results.

It is worth mentioning that the above method requires very little computing time. For example, the sample problem required 2.5 h when the finite element method based on finite strain theory was used whereas the control volume method required only 150 s. Both computations were performed on CDC Cyber. The computational saving achieved by this method will further facilitate the use of computer models for on-line control of rolling processes.

CONCLUSIONS

The results presented in this report, though preliminary in nature, have demonstrated that the model is capable of predicting inhomogeneous deformation in a rolling process. The coupled thermal/mechanical behaviour, including the heat generation from plastic energy dissipation, is appropriately accounted for. The relatively small amount of computing time required makes possible its application to on-line control of rolling processes, using a micro-computer.

Currently, the model can compute temperature and velocity distributions under a steady-state condition. Further work is required to implement strain calculations using velocity results. Extension to a 3-dimensional model and to account for transient conditions can be achieved with some modification to the formulations.

The model, though specifically developed for the modelling of rolling processes, should be equally applicable to the analyses of extrusion and rapid cooling processes by making minor modifications.

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Table 1 - Predicted temperatures at nodal points - with and without the consideration of heat generated by plastic deformation

y/h		X (distance from entry point)				
		0	24.68	37.02	49.36	61.7 exit
0 centre	with DQ	1200.713	1202.668	1203.333	1203.961	1201.902
	no DQ	1200.0	1200.0	1200.0	1200.0	1197.359
0.35	with DQ	1199.498	1193.878	1189.313	1183.912	1178.066
	no DQ	1198.441	1185.006	1175.821	1166.6	1159.205
0.45	with DQ	1137.128	1165.123	1145.016	1121.785	1108.546
	no DQ	1134.995	1130.389	1095.815	1070.169	1056.541
0.5 surface	with DQ	870.669	888.545	875.711	860.860	852.191
	no DQ	869.307	866.367	844.298	827.951	819.170

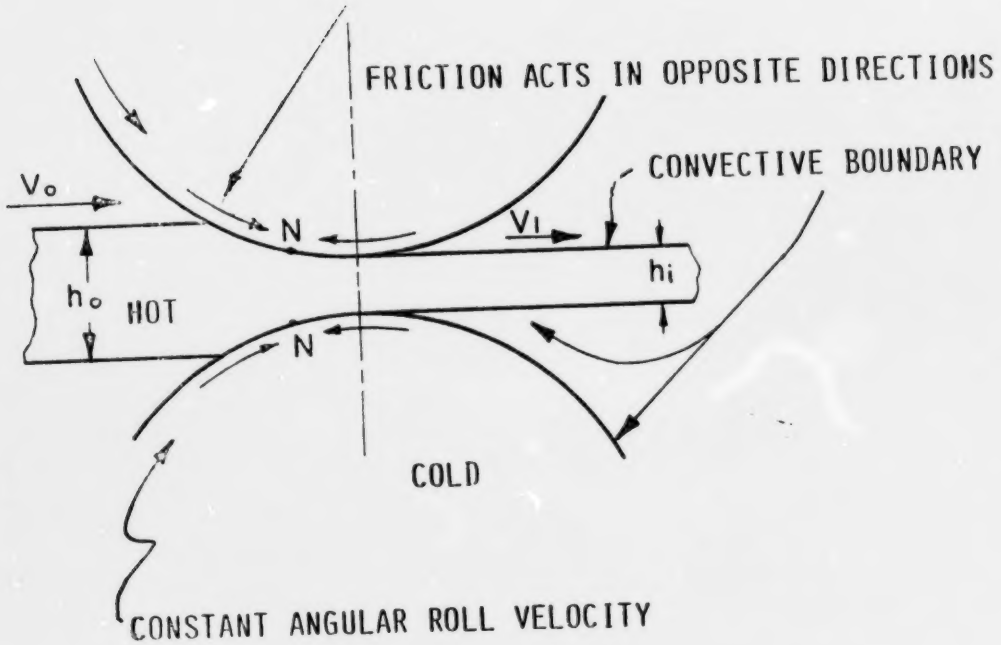


FIG. 1 SCHEMATIC OF ROLLING

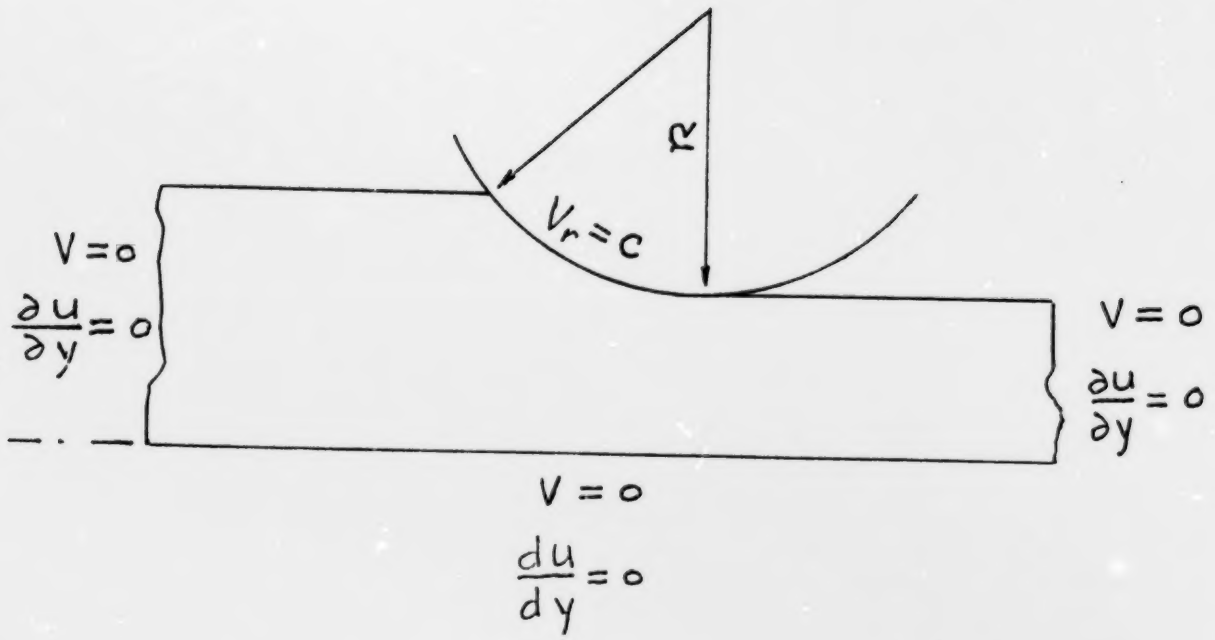


FIG. 2 BOUNDARY CONDITIONS OF AN INCOMPRESSIBLE FLOW PROBLEM

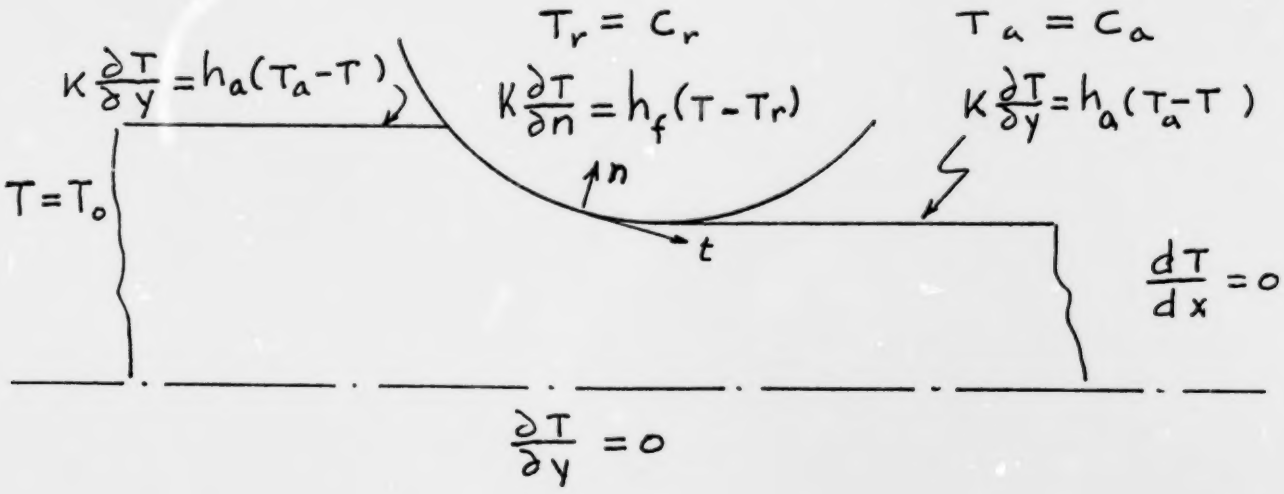

$T_r = C_r$
$T_a = C_a$
$K\frac{\partial T}{\partial y} = h_a(T_a - T)$
$K\frac{\partial T}{\partial n} = h_f(T - T_r)$
$K\frac{\partial T}{\partial y} = h_a(T_a - T)$
n
t
$T = T_o$
$\frac{dT}{dx} = o$
$\frac{\partial T}{\partial y} = o$

FIG. 3. BOUNDARY CONDITIONS OF A THERMAL DIFFUSION-CONVECTION PROBLEM.

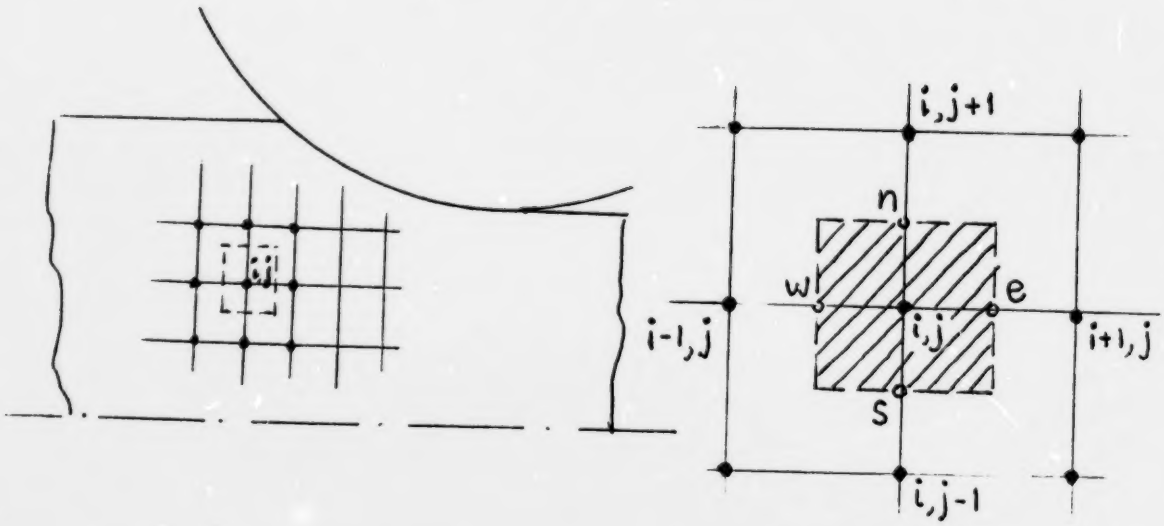
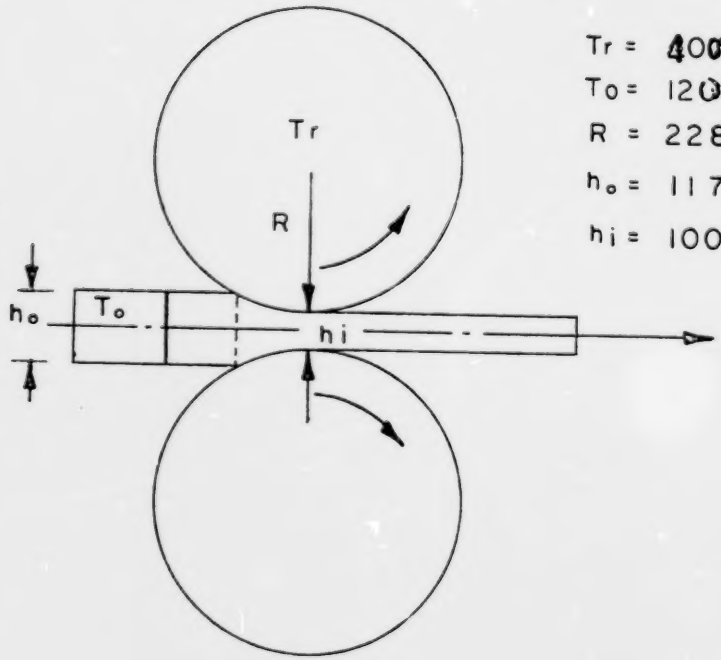


FIG.4 CONTROL VOLUME DISCRETIZATION



Tr = 400°C
To = 1200 °C
R = 228.5 mm
h_o = 117.0 mm
h_i = 100.3 mm

FIG.5 GEOMETRICAL DIMENSIONS OF THE WORKPIECE AND THE ROLLS AND INITIAL TEMPERATURES

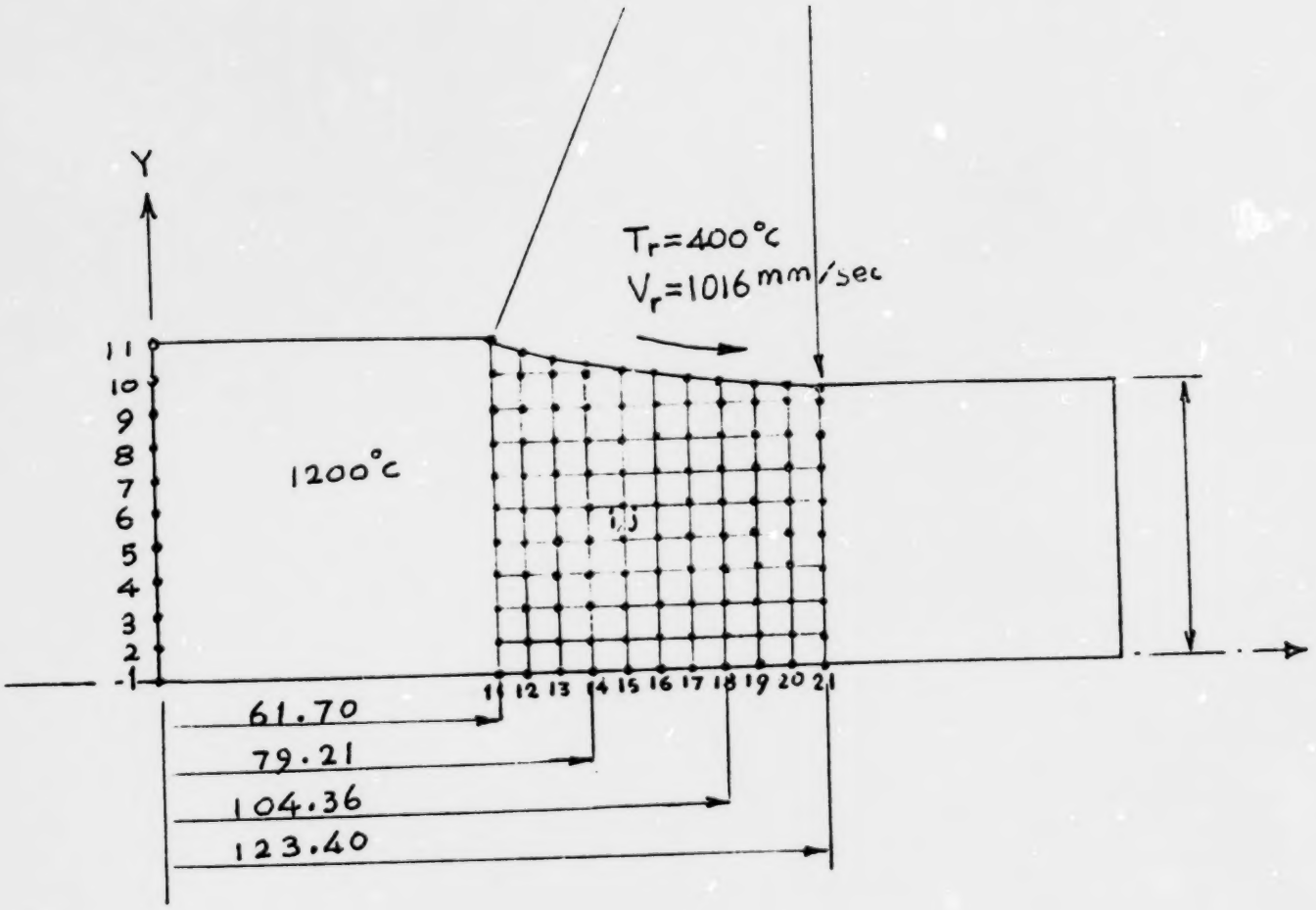


FIG. 6 DISCRETIZED MESH POINTS FOR A CONTROL VOLUME ANALYSIS

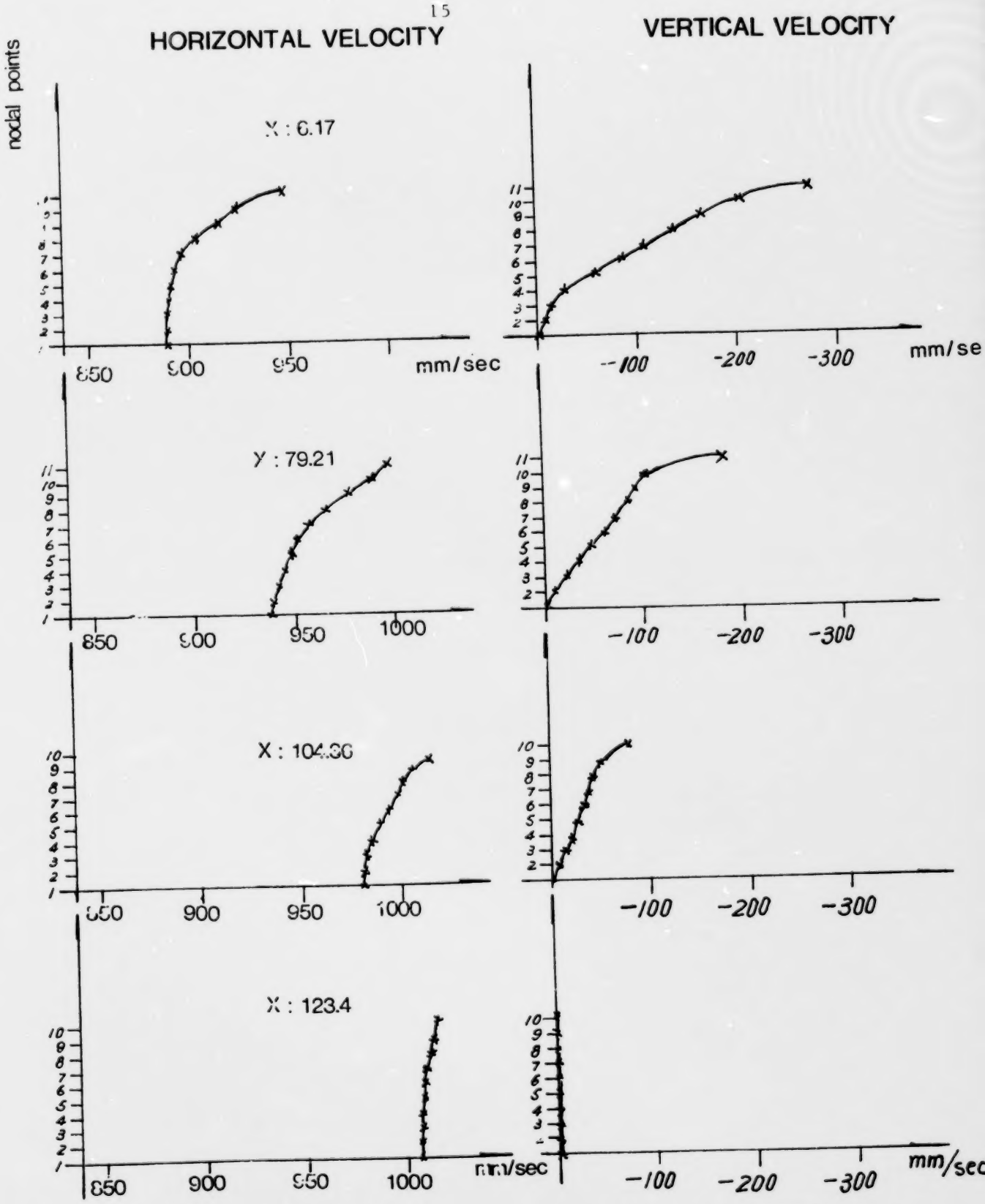


FIG. 7 HORIZONTAL AND VERTICAL VELOCITY PROFILES (STICKING CONDITION)